Scaling Up Distance Labeling on Graphs with Core-Periphery Properties

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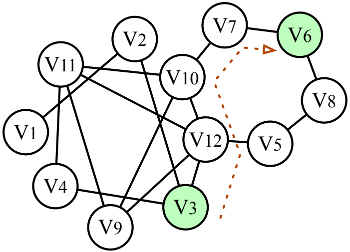
Problem

Given a graph $G(V,E)$ and two nodes $s,t \in V$, report the length of the shortest path from s to t



Solutions

- Online search such as Breadth-first search (BFS)
- Index-based approaches such as 2-hop labeling
  - Each node v has a label L_v
    - » $L_{v_3} = \{v_1 : 2, v_2 : 1, v_3 : 0\}$
  - Label size is denoted as $|L_v|$, e.g., $|L_{v_3}| = 3$
  - Index size is computed as $\Sigma_{v \in V}|L_v|$

Observations on Index Size

Graphs with larger # nodes/edges do not mean larger index

- DBLP ($n = 1.3M$) with $51G$ index
- INDO ($n = 7.4M$) with $17.7G$ index

Index of social networks are normally larger than road networks

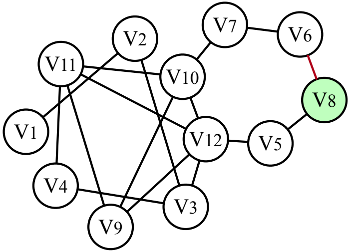
- BELG ($n = 1.4M$) with $4.4G$ index

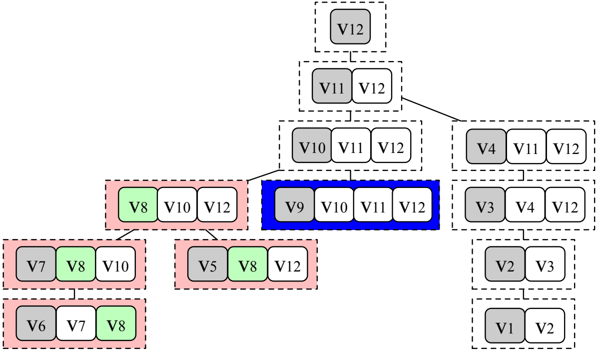
Index Size vs. Treewidth

Explanations from Treewidth

A tree decomposition T of a graph G is to convert G into a tree, where each tree vertex (i.e., bag) contains a subset of nodes in G

- Each node appears in at least one bag
- Each edge is contained in at least one bag
- Each node induces a subtree





- The width of a tree decomposition T is its maximal bag size, e.g. width $= 4$
- The treewidth of G is the minimum width over all tree decompositions of G

Explanations from Treewidth

The treewidth of G is the minimum width over all tree decompositions

Contribution #1

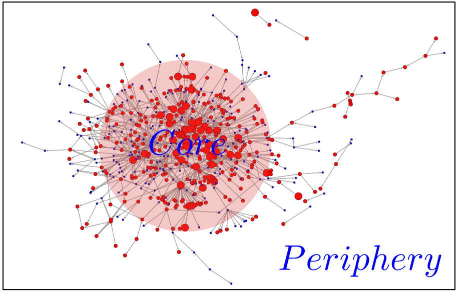
Given a graph with n nodes and treewidth tw, the index size generated by the best 2-hop labeling algorithm is $\tilde{\Theta}(n \cdot tw)$ in the worst-case.

- 2-hop labeling properly handles graphs with relative low treewidth, e.g., road networks
- 2-hop labeling *fails* for the oversized index on graphs with relatively large treewidth, e.g., social networks and web graphs
  - The index size on UK07 exceeds 500G

Core Periphery Structure

2-hop labeling fails for the oversized index on graphs with relatively large treewidth.
The core-periphery structure has been identified in real graphs

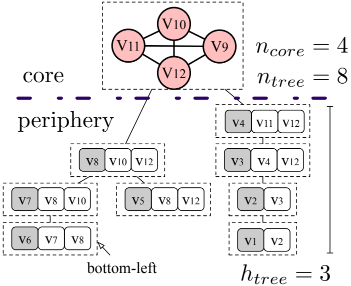
- A densely connected core
- The other nodes, called the periphery, are of limited connectivity



Core Tree Decomposition and Our Contributions

Core Tree Decomposition[1] is a tree decomposition with parameter d

- One big bag for core part (with bag size $> d$)
  - n_{core} be the number of nodes (in G) that appear in the core
- Many small bags for periphery part (with bag size $\leq d$)
  - n_{tree} be the number of bags in the periphery
  - h_{tree} be the maximum height of trees in the periphery
  - w be the width of tree decomposition after decomposing the core



Core Tree Decomposition ($d = 3$)

Contribution #2 (Index Size)

$\tilde{O}((n_{core} + n_{tree}) \cdot w) \rightarrow \tilde{O}(n_{core} \cdot w) + O(n_{tree} \cdot (d + h_{tree}))$

- 2-hop labeling on core
- Tree index on periphery ($w \rightarrow d + h_{tree}$)

Contribution #3 (Tree Index Time)

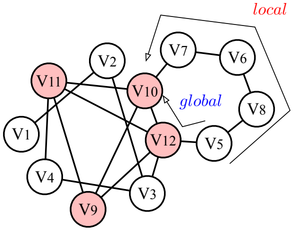
$O(n_{tree} \cdot m) \rightarrow O(n_{tree} \cdot d(d + h_{tree}))$

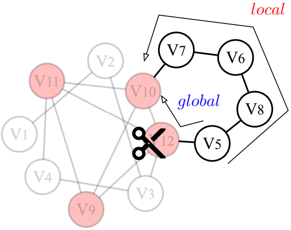
- n_{tree} BFSs, each with cost $O(m)$

[1] Takuya, Akiba, et al. Shortest-path queries for complex networks: exploiting low tree-width outside the core

Local Distances

- Local distance from s to t is the minimized length of paths from s to t not via any node in the core
  - Path # 1, $< v_5, v_{12}, v_{10} >$, contains v_{12} in the core ☹
  - Path # 2, $< v_5, v_8, v_6, v_7, v_{10} >$, does not via nodes in the core ☺
- To *avoid* exploring the whole graph to compute the distances
- Local distances are sufficient for efficiently computing distance queries





Experiments

Algorithms

- PSL^+ and PSL^* (2-hop labeling[2])
- CT-Index, the proposed algorithm ($d = 100$)

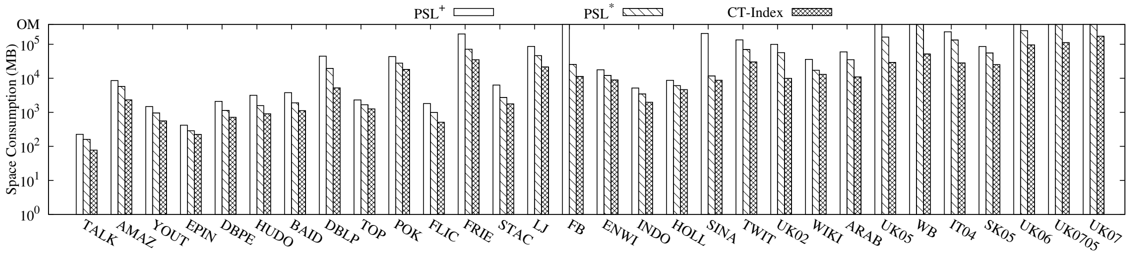
Dataset

- 30 real graphs
  - Including social networks, web graphs, coauthorship graphs, communication networks, and interaction networks
- The largest graph has over 5.5 billion edges

[2] Wentao, Li, et al. Scaling distance labeling on small-world networks

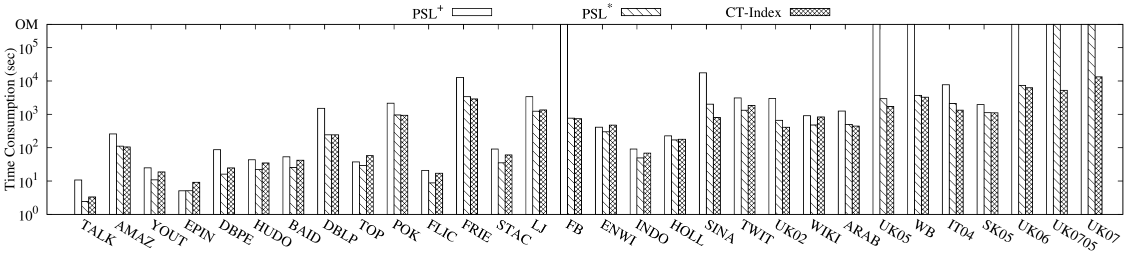
The Comparison of the Index Size

- CT-Index can index massive graphs such as UK0705 and UK07
- CT-Index vs. PSL^+: reduces 4.79 on average, 23.72 at a maximum
- CT-Index vs. PSL^*: reduces 2.31 on average, 5.66 at a maximum

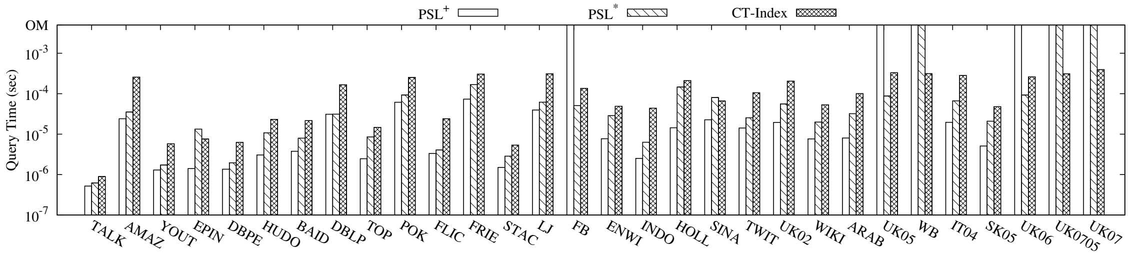


The Comparison of the Index Time

- CT-Index shortens the index time on most graphs
- CT-Index vs. PSL$^+$: reduces 3.26 on average, 21.85 at a maximum
- CT-Index vs. PSL*: reduces 1.68 on average, 4.64 at a maximum



- CT-Index vs. PSL^{+}: 7.55 times slower on average
- CT-Index vs. PSL^{*}: 3.17 times slower on average
- Below 0.4 milliseconds including on UK07 with 5.5 billion edges



Summary

- Limitation of 2-hop labeling on graphs with relative high treewidth
- Core tree decomposition for smaller index size
- Local distances for efficient tree index construction

Thank You